

## Corrigendum<sup>1</sup>

When counting lines, statement and proof of Theorems, Propositions, Corollaries or Lemmas are considered as one text, and diagrams (but not sequences split over several lines) are considered as one line.

**(I.1.2)** Delete the paragraph after (I.1.2.3).

**(I.1.2.4)** In lines 2 and 5, delete “a)” (2 times); delete statement b) and the proof of statement b).

**(I.1.3.16)** In line 5, replace  $R[N]$  by  $R[M \setminus P]$ .

**(I.1.4.5)** In line 1, replace “over” by “under”; in line 4, replace “under” by “over”; in line 7, replace  $\text{Sch}/R$  by  $\text{Sch}/F(R)$ .

**(I.1.4.9)** In line 13, insert “,  $S$  is a scheme,” after “category”.

**(I.1.4.15)** In line 6, replace  $X_{M,\omega}(R)$  by  $X_M(R)$ .

**(I.2.4.2)** In line 6, delete “a)”.

**(I.2.4.3)** In line 4, delete “, or  $I = \emptyset$ ”; delete the first two sentences of the proof.

**(I.2.5.1)** In line 1, replace “cancellable, finitely generated” by “integrally closed”; in line 4, replace “Since  $M$  is integrally closed by 1.2.4 b), this” by “This”.

**(I.2.5.2)** In line 1, replace “cancellable, finitely generated” by “integrally closed”.

**(I.2.5.3)** In line 3, replace “, cancellable and finitely generated” by “and integrally closed”.

**(I.2.5.4)** Delete this completely.

**(I.2.6.7)** In line 3, delete “ $I \neq \emptyset$ ”.

**(I.2.6.10)** In line 5, delete “, or  $I = \emptyset$ ”.

**(II.1.1.8)** In line 8, replace “Then,  $W \cap V'$ ” by “If moreover the  $R$ -module  $W \cap V'$  is free (for example, if  $R$  is principal), then it”; in line 15, replace “and  $W'$  is the induced  $R$ -structure on  $V'$ ” by “such that the  $R$ -modules  $W' := W \cap V'$  and  $W/W'$  are free (for example, if  $R$  is principal)”; in line 16, delete “the  $R$ -module”.

**(II.1.1.9)** In line 9, replace “and  $W'$  the induced  $R$ -structure on  $V'$ ” by “such that the  $R$ -module  $W' := W \cap V'$  is free”; in line 11, insert “if moreover the  $R$ -module  $W/W'$  is free” before “the canonical projection”.

**(II.1.1.11)** In line 10, replace “and  $W_i$  is the induced  $R$ -structure on  $V_i$ ” by “such that the  $R$ -module  $W_i := W \cap V_i$  is free”; in line 18 insert “such that the  $R$ -module  $W \cap V'$  is free” after “of  $V$ ”.

**(II.1.1.15)** In line 1, replace “Let  $V$ ” by “Let  $R \subseteq \mathbb{R}$  be a subring, let  $V$ ”; in line 7, insert “suppose that  $R$  is principal,” after “Now,”.

**(II.1.4.1)** In line 3, insert “finitely many” after “intersections of”, and insert “such that the  $R$ -module  $W \cap \langle A \rangle$  is free, and if  $A$  is moreover” after “If  $A \subseteq V$  is”.

**(II.1.4.13)** In line 2, insert “and that the  $R$ -modules  $W \cap U$  and  $W/W \cap U$  are free” after “ $W$ -rational”.

**(II.1.4.14)** In line 2, insert “and that the  $R$ -modules  $W \cap \langle A \cap B \rangle$  and  $W/W \cap \langle A \cap B \rangle$  are free” after “not full”.

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(II.1.4) In line 4 of the paragraph before (II.1.4.15), replace “face” by “faces”.

(II.1.4.29) In line 1, replace “ $W$ -polycone” by “polycone”; in line 8, replace “the canonical ordering on  $\langle A \rangle$ ” by “the ordering on  $\langle A \rangle$  defined by  $A$ ”; in line 12, replace “the canonical ordering on  $\langle x \rangle$ ” by “the ordering on  $\langle x \rangle$  defined by  $\text{cone}(x)$ ”; in line 13, delete “again”.

(II.1.4.31) In line 26, replace [A, VII.4.3 Lemme 1] by [A, VII.4.2 Lemme 1].

(II.1.4.33) In line 4, replace  $R_{>0}$  by  $R$ .

(II.1.5.1) In line 2, replace “denote by  $W_i$  the  $R$ -structure induced by  $W$  on  $V_i$ ” by “suppose that the  $R$ -module  $W_i := W \cap V_i$  is free”.

(II.1.5.3) At the beginning of the proof, insert “We can suppose that  $A$  is full.”.

(II.2.2.5) In line 1, delete “and  $W_\sigma := W/\langle \sigma \rangle$ ”; at the end, add “If the  $R$ -modules  $W \cap \langle \sigma \rangle$  and  $W/W \cap \langle \sigma \rangle$  are free, then we set  $W_\sigma := W/\langle \sigma \rangle$ .”.

(II.2.2.6) In line 1, replace “and let  $\sigma \in \Sigma$ ” by “let  $\sigma \in \Sigma$ , and suppose that the  $R$ -modules  $W \cap \langle \sigma \rangle$  and  $W/W \cap \langle \sigma \rangle$  are free”.

(II.2.2.11) In line 1, insert “, and suppose that the  $R$ -modules  $W \cap \langle s(\Sigma) \rangle$  and  $W/W \cap \langle s(\Sigma) \rangle$  are free” after “in  $V$ ”.

(II.2.2.12) In line 1, insert “, and suppose that the  $R$ -modules  $W \cap \langle s(\Sigma) \rangle$  and  $W/W \cap \langle s(\Sigma) \rangle$  are free” after “in  $V$ ”.

(II.2.3.6) In line 1, insert “convex” before “neighbourhood”; replace the first sentence of the proof by “First we can suppose without loss of generality that every cone in  $\Sigma$  met by  $U$  contains  $x$ , and then on use of 1.2.18 and 1.2.19 we can suppose without loss of generality that every  $\sigma \in \Sigma_{<n}$  containing  $x$  is contained in  $A$ .”; in line 18, replace “, contradicting” by “. As  $U$  is convex it holds  $\llbracket x, y \rrbracket \subseteq U$ , hence  $\tau$  meets  $U$ , therefore contains  $x$  and thus is contained in  $A$ . This contradicts”.

(II.2.3.20) In line 1, replace “ $\sigma \subseteq \text{in}(|\Sigma|)$ ” by “ $\sigma \cap \text{in}(|\Sigma|) \neq \emptyset$ ”.

(II.3.2.1) In line 2, insert “sharp and” before “free over  $\Sigma$ ”.

(II.3.4.3) In line 1, insert “, and suppose that  $R$  is principal” after “in  $V$ ”.

(II.3.6) In line 1 of the general hypotheses, replace “and let  $\xi \in \Sigma_1$ ” by “let  $\xi \in \Sigma_1$ , and suppose that the  $R$ -modules  $W \cap \langle \xi \rangle$  and  $W/W \cap \langle \xi \rangle$  are free”.

(II.3.7.3) At the beginning of the proof, insert “Without loss of generality we may suppose that  $R = K$ .”.

(II.3.7.5) At the beginning of the proof, insert “Without loss of generality we may suppose that  $R = K$ .”.

(II.4.1.1) In line 2, replace  $(m_\sigma + \sigma)_{\sigma \in \Sigma}$  by  $(m_\sigma + \sigma^\vee)_{\sigma \in \Sigma}$ .

(II.4.1.5) In line 18, replace  $m - \rho$  by  $m_\rho$ .

(II.4.1.13) In line 4, replace “exists” by “exist”.

(II.4.2.6) In line 4, replace 4.2.4 by 4.2.3.

(II.4.2.7) In line 2, delete “1-dimensional”.

(II.4.3.1) In line 2, replace “cancellable, finitely generated” by “finitely generated, integrally closed”; in line 5, replace  $A \cap M$  by  $A^\vee \cap M$ ; in lines 6 and 16, replace  $V$  by  $V^*$ ; in line 19, replace  $Y \cap Ms$  by  $Y \cap M$ ; in line 19, replace “the claim is proven” by “ $A^\vee \cap M$  is finitely generated”; at the end of the proof, add the following paragraph: “Finally, let  $x \in \text{Diff}(A^\vee \cap M) = \langle A^\vee \cap M \rangle_{\mathbb{Z}} \subseteq M$  and

let  $m \in \mathbb{N}$  such that  $mx \in A^\vee \cap M$ . As  $A^\vee$  is conic it follows  $x \in A^\vee \cap M$ , and therefore  $A^\vee \cap M$  is integrally closed.”

**(II.4.3.3)** In line 2, replace “. Then, there exists” by “, and let”, and “, that  $B^\vee \cap M = (A^\vee \cap M) \oplus \mathbb{N}_0(-u)$ , that” by “. Then, it holds  $B^\vee \cap M = A^\vee \cap M - \mathbb{N}_0 u$  and”; in line 4, delete “that”; in line 6, replace “ $u = 0$  fulfils the claim” by “this is clear”; delete the third sentence of the proof; delete lines 17–19.

**(II.4.3.4)** In line 2, replace  $\cup$  by  $+$ .

**(II.4.3.5)** In line 6, replace “cancellable and finitely generated” by “finitely generated and integrally closed”.

**(III.1.3.2)** In line 18, insert “if  $R$  is an  $H$ -graded ring, then” after “other hand,”; in lines 18–20, replace  $g$  by  $h$  (7 times),  $h$  by  $g$  (2 times), and  $G$  by  $H$  (2 times); in line 21, replace  $\text{GrAnn}^G$  by  $\text{GrAnn}^H$ .

**(III.1.3.5)** In line 26, insert “if  $M$  is an  $H$ -graded  $R_{[\psi]}$ -module, then” after “other hand,”; in lines 26–29, replace  $g$  by  $h$  (7 times),  $h$  by  $g$  (2 times), and  $G$  by  $H$  (2 times); in line 31, replace  $\text{GrMod}^G(R)$  by  $\text{GrMod}^H(R_{[\psi]})$ .

**(III.2.4.1)** In line 11, replace  $L_g$  by  $M_g$ .

**(III.2.4.6)** In line 4, replace “is” by “are”.

**(III.3.4.7)** In line 5, replace “if” by “it”

**(III.3.4.9)** In lines 9–10, replace  $g$  by 0 (4 times); in line 15, replace  $d$  by 0.

**(III.3.6.2)** Delete the last sentence.

**(III.3.6.5)** In line 26, replace  $\mathfrak{a}_k e$  by  $\mathfrak{a}_k e = 0$ .

**(III.4.3.1)** In line 11, replace “these  $\delta$ -functors are universal” by “the  $\delta$ -functors  $({}^G\text{Ext}_R^i(\bullet, M))_{i \in \mathbb{N}_0}$  and  $({}^G\text{Ext}_R^i(M, \bullet))_{i \in \mathbb{N}_0}$  are universal for every  $G$ -graded  $R$ -module  $M$ ”.

**(III.4.3.2)** In line 4, replace  $\mathbb{Z}$  by  $\mathbb{N}_0$ , and replace “, namely” by “and that  $({}^G\text{Ext}_R^i(M, \bullet)_{[\psi]})_{i \in \mathbb{Z}}$  is”.

**(III.4.3.3)** In line 4, replace  $({}^G\text{Ext}_R^i(\bullet, M)_{[\psi]})_{i \in \mathbb{Z}}$  by  $({}^G\text{Ext}_R^i(\bullet, M)_{[\psi]})_{i \in \mathbb{N}_0}$ ; in line 5, replace “, namely” by “and that  $({}^G\text{Ext}_R^i(\bullet, M)_{[\psi]})_{i \in \mathbb{Z}}$  is”; in line 9, replace  $\mathbb{Z}$  by  $\mathbb{N}_0$ ; in line 10, replace “, namely” by “and that  $({}^H\text{Ext}_{R_{[\psi]}}^i(\bullet_{[\psi]}, M_{[\psi]}))_{i \in \mathbb{Z}}$  is”.

**(III.4.3.8)** In line 17, replace  $\mathbb{Z}$  by  $\mathbb{N}_0$ .

**(III.4.3.9)** In line 8, replace  $\mathbb{Z}$  by  $\mathbb{N}_0$ .

**(III.4.3.10)** In line 9, replace  $\mathbb{Z}$  by  $\mathbb{N}_0$ ; in line 12, replace “latter functor” by “functor on the left hand side”.

**(III.4.4.1)** In line 6, replace “ $({}^G H_{\mathfrak{A}}^i)_{i \in \mathbb{Z}}$  and  $({}^G D_{\mathfrak{A}}^i)_{i \in \mathbb{Z}}$ ” by “ $({}^G H_{\mathfrak{A}}^i)_{i \in \mathbb{N}_0}$  and  $({}^G D_{\mathfrak{A}}^i)_{i \in \mathbb{N}_0}$ ”.

**(III.4.4.7)** In line 17, replace  $M$  by  $\text{Id}_{\text{GrMod}^G(R)}(\bullet)$ ; in line 25, replace  $\mathbb{Z}$  by  $\mathbb{N}_0$ .

**(III.4.4.12)** In line 2, replace “a  $\delta$ -functor” by “an exact  $\delta$ -functor”.

**(III.4.5.12)** In line 10, replace  ${}^G C(\mathfrak{a}, I) = 0$  by  ${}^G C(\mathfrak{a}, I) = I$ , and replace  $\mathbb{Z}$  by  $\mathbb{N}$ ; in line 14, replace “ $\text{CCo}(\text{GrMod}^G(R))_{\dots}$ ” by “ $\text{CCo}(\text{GrMod}^G(R))_{\dots}$ ”; in line 17, replace “ $i > 1$ .” by “ $i > 1$ , hence in particular  ${}^G H^{i-1}(\mathfrak{b}, I_{a_n}) \cong {}^G H^i(\mathfrak{a}, I)$  for  $i > 1$ .”; replace lines 18–19 by “Since  ${}^G \Gamma_{\langle a_n \rangle R} \circ \text{Ker}(\eta_{a_n}) = \text{Ker}(\eta_{a_n})$  and  ${}^G \Gamma_{\langle a_n \rangle R} \circ \text{Coker}(\eta_{a_n}) = \text{Coker}(\eta_{a_n})$  it follows from 4.4.17 and 4.4.7 a) that there is an exact sequence”; in line 29, replace  $\eta$  by  $\eta_{a_n}$ .

d

**(IV.1.1.7)** In line 3, replace  $E$  by  $F$ ; in line 4, replace “We denote” by “For a ring  $R$ , we denote by  $R[(Y_f)_{f \in F}]$  the polynomial algebra in the indeterminates  $(Y_f)_{f \in F}$  over  $R$ , and”; in lines 9 and 12, replace “over” by “under”.

**(IV.1.3.2)** In line 6, indent “if  $x \notin \sigma^\perp$ ”; in line 11, replace “form” by “from”; in line 17, delete “on use of I.1.3.16”; in line 18, reverse both vertical arrows.

**(IV.1.3.3)** In lines 6, 11 and 15, replace  $\tau_M^\vee \oplus \mathbb{N}_0(-u)$  by  $\tau_M^\vee - \mathbb{N}_0 u$ ; in lines 6 and 13, replace  $(\tau_M^\vee \cap \sigma^\perp) \oplus \mathbb{N}_0(-u)$  by  $\tau_M^\vee \cap \sigma^\perp - \mathbb{N}_0 u$ ; in line 10, replace I.1.3.8 by I.1.3.14; replace line 12 by “ $R[\tau_M^\vee \cap \sigma^\perp] \otimes_{R[\tau_M^\vee]} R[\tau_M^\vee]_{e_u} \cong R[\tau_M^\vee \cap \sigma^\perp]_{e_u} \cong$ ”.

**(IV.1.3.4)** In lines 7, 11 and 13, and in the lower row of the diagram on line 9, replace  $\tau_M^\vee \oplus \mathbb{N}_0(-u)$  by  $\tau_M^\vee - \mathbb{N}_0 u$ .

**(IV.1.3.6)** In line 2, insert “injective” before “open”; in line 3, replace “properties” by “property”.

**(IV.1.3.13)** In line 1, replace “exists” by “exist”.

**(IV.2.1.3)** In line 12, replace  $\bullet[\text{Ker}(a^+)]_{(B)} = \bullet[\text{Im}(c) \cap \mathbb{N}_0^{\Sigma_1}]_{(B)}$  by  $\bullet[\text{Ker}(a^+)] = \bullet[\text{Im}(c) \cap \mathbb{N}_0^{\Sigma_1}]$ .

**(IV.2.1.4)** In line 7, replace  $S_{\Sigma, B}(R)$  by  $S_{\Sigma_1, B}(R)$ ; in line 8, replace  $\Sigma$  by  $\Sigma_1$ ; in line 10, replace  $\mathbb{T}_{R, \Sigma, B}$  by  $\mathbb{T}_{R, \Sigma_1, B}$ .

**(IV.2.1.6)** In line 4, replace  $S_{\Sigma, B}(R)$  by  $S_{\Sigma_1, B}(R)$ .

**(IV.2.1.11)** In line 9, replace  $S_{\Sigma, B}(R)$  by  $S_{\Sigma_1, B}(R)$  (2 times); in line 11, replace  $\mathbb{T}_{R, \Sigma, B}$  by  $\mathbb{T}_{R, \Sigma_1, B}$ .

**(IV.2.3.3)** Replace the second and the third sentence of the proof by the following: “Therefore, if  $\Sigma$  is full then  $c_\sigma$  is an isomorphism for  $\sigma \in \Sigma$ , hence so is  $\gamma_\sigma$  for  $\sigma \in \Sigma$ , and thus so is  $\gamma_\Sigma$ . Conversely, suppose that  $\gamma_\Sigma$  is an isomorphism and let  $\sigma \in \Sigma$ . The (topological) image of  $Y_\sigma(\mathbb{Z})$  under  $\gamma_\sigma(\mathbb{Z})$  is nonempty (I.1.4.11), closed in  $X_\sigma(\mathbb{Z})$  since  $\gamma_\sigma(\mathbb{Z})$  is a closed immersion (2.3.1), and open in  $X_\sigma(\mathbb{Z})$  since  $\gamma_\Sigma$  is an isomorphism and hence open. As  $X_\sigma(\mathbb{Z})$  is connected (1.2.2) this implies that  $\gamma_\sigma(\mathbb{Z})$  is surjective and therefore an epimorphism in the category of affine schemes (as can be seen on use of [ÉGA I.5.2.5]). Therefore,  $\mathbb{Z}[c_\sigma]$  is a monomorphism, hence so is  $c_\sigma$  (I.1.3.13), and thus the claim is proven.”

**(IV.3.1.3)** In line 2, replace  $\text{GrMod}^B(S)$  by  $\text{GrMod}^B(S_{(B)})$ .

**(IV.3.1.14)** In line 2, replace “graded” by “ $P$ -graded”.

**(IV.3.2.6)** In line 17, replace  $\eta_\alpha$  by  $\eta_{\Sigma, \alpha}$  (2 times).

**(IV.3.3.3)** In line 27, replace  $g'_\tau \upharpoonright_{Y_{\tau \cap \sigma} \upharpoonright_{Y_{\tau \cap \sigma \cap \tau'}}} - g'_{\tau'} \upharpoonright_{Y_{\tau' \cap \sigma} \upharpoonright_{Y_{\tau' \cap \sigma \cap \tau}}}$  by

$$g'_\tau \upharpoonright_{Y_{\tau \cap \sigma} \upharpoonright_{Y_{\tau \cap \sigma \cap \tau'}}} - g'_{\tau'} \upharpoonright_{Y_{\tau' \cap \sigma} \upharpoonright_{Y_{\tau' \cap \sigma \cap \tau}}}.$$

**(IV.3.4.1)** In line 5, replace  $\mathfrak{a}^{\text{sat}, I_N}$  by  $\mathfrak{a}^{\text{sat}, I_B}$ .

**(IV.3.4.2)** In line 12, insert “for every  $\sigma \in \Sigma$ ” after  $\mathfrak{a}_{(\sigma)} = \mathfrak{b}_{(\sigma)}$ .

**(IV.4.1.1)** In line 12, replace  $\mathcal{O}_{Y_\Sigma(U)}$  by  $\mathcal{O}_{Y_\Sigma(U)}$ .

**(IV.4.1.5)** In line 7, replace  $\text{Mod}(S_{(\sigma)})$  by  $\text{Hom}(\text{GrMod}^B(S_{(B)}), \text{Mod}(S_{(\sigma)}))$ .

**(IV.4.1.6)** Replace everything except the first sentence by “A counterexample can be obtained by considering the Cox scheme over  $\mathbb{Z}$  associated with the complete  $\mathbb{Z}^2$ -fan  $\Sigma$  in  $\mathbb{R}^2$  with maximal cones  $\text{cone}((1, 0), (0, 1))$ ,  $\text{cone}((0, 1), (-2, -3))$  and  $\text{cone}((-2, -3), (1, 0))$ ; its Cox ring is  $S = \mathbb{Z}[Z_1, Z_2, Z_3]$  with  $\deg(Z_1) = 1$ ,  $\deg(Z_2) = 2$  and  $\deg(Z_3) = 3$ , where  $A = \mathbb{Z}$ .”

**(IV.4.2.1)** In line 8, replace  $\mathcal{S}_\Sigma(F(\alpha))$  by  $\mathcal{S}_{\Sigma, B}(F(\alpha))$ .

**(IV.4.2.4)** Delete the last sentence.

**(IV.4.2.5)** In line 7, replace “this is equivalent to  ${}^A\Gamma_I(\text{Ker}(\bar{\eta}_{\Sigma,B}(F^{(A)}))) = \text{Ker}(\bar{\eta}_{\Sigma}(F^{(A)}))$  and  ${}^A\Gamma_I(\text{Coker}(\bar{\eta}_{\Sigma,B}(F^{(A)}))) = \text{Coker}(\bar{\eta}_{\Sigma}(F^{(A)}))$ , and hence” by “and exactness of  $\bullet_{(B)}$ ”.

**(IV.4.2.8)** In line 28, replace  $y$  by  $x$ .

**(IV.4.2.13)** In line 7, replace “ $H_I^2(S)_0 = H_I^2(S) \cong H_{IS_I}^2(S)$  is not finitely generated.” by “ $H_I^2(S)_I \cong H_{IS_I}^2(S_I)$  is not finitely generated, and hence the  $R$ -vector space  ${}^A H_I^2(S)_0 = H_I^2(S)$  is not finitely generated, too.”.

**(Logical Bibliography)** In [2], replace “applications” by “applications”.

**(Index of Notations)** In column 4, replace “ $A_k$  ( $A$  convex)” by “ $A_k$  ( $A$  conic)”.